Analysis of Identical Parallel System with Failure followed by Inspection Policy using Discrete Distribution

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Abstract - In this paper two identical operative parallel units are analyzed probabilistically using regenerative point techniques. A single repair facility is available for repair of failed unit after being followed by inspection policy.The distribution of failure; inspection and repair time are taken as discrete distribution. Various important measures of system reliability like mean time to system failure (MTSF), steady state availability, profit function and busy period of repairman and inspection are obtained. The profit function and MTSF has been verified by plotting graphs.

Keywords: Geometric distribution, Regenerating point technique, MTSF, Availability, Busy period and Profit function.

1. Introduction

In modern discipline the development of science and technology and the needs of modern society are racing against each other. Therefore, industries are trying to introduce more and more automation in their industrial processes in order to meet the ever increasing demands of society and accordingly the complexity of industial system are increasing one by one. This rise the concept of reliability which deals with the development of new techniques for increasing system effectiveness by reducing frequency of failure and minimizing the maintenance cost.In the field of reliability many researchers had analyzed reliability models in which the life time and repair time distribution were taken as continous distribution.Said, Salah, Sherbeny[6] had analyzed two unit cold standby system with preventive maintenance and random change in units.By this paper he introduced the concept of inspection to check the repair mechanism being satisfactory or not.Taneja[7], Goel[8], Naidu[9] had also analyzed reliability models by indroducing the concept of inspection using continous distribution. In all these papers one thing is common that the observed data was found to be large.But this was not true in case of small data. In such cases, the discrete failure time distributions are considered to be appropriate one as compaired to continous distributions.

Bhardwaj, Gupta had given their contribution in the area of reliability using discrete distribution by analyzing parallel systems with Geometric failure and repair time distributions.Bhardwaj[1] had analyzed two unit redundant systems with imperfect switching and connection time. In his research he analyzed two identical unit standby and parallel system with two types of failure and repair time. Gupta [2] had also analyzed the two identical unit parallel system with geometric failure and repair time distributions.In all these research no one has given any consideration for the inspection of failure. Keeping this in mind we had analyzed in this paper the two identical operative parallel systems by introducing the concept of inspection policy.The inspection and repair time are taken as geometric distribution. Initially both the automatic units are in operative condition.On the failure of an automatic unit, an inspection is being performed before being repaired by the repairman.

The model is analysed stochastically and the expressions for the various reliability measures of system effectiveness such as MTSF, steady state availability, and busy period for both inspector and repairman were obtained.Graphs were also been drawn to analysed the behavior of MTSF and profit function with respect to repair and failure rate.

2. Model description

The following assumptions are associated with the model:

- A system consists of two identical units arranged in a parallel network. Initially one automatic unit is in operative condition and the other is in cold standby.
- Upon the failure of an automatic unit, the cold standby unit becomes operative instantaneously.
- The system is assumed to be in the failed state when both units together were in failed conditions.
- Inspection policy is being introduced for inspecting the failed automatic unit.
- A single repairman is available to repair the failed unit.
- A repaired unit's works as good as new.

3 Nomenclature

O	:	Unit is in operative mode
S	:	Unit is in standby mode
A_o	:	Automatic unit is in operative mode.

A_i / A_{iw}	:	Automatic unit is in failure mode and under inspection / waiting for inspection.
A_r / A_{rw}	:	Automatic unit is in failure mode and under repair /waiting for repair.
p_1 / q_1	:	Probability that automatic unit goes to failed state.
p_2 / q_2	:	Probability of the failed unit to be inspected satisfactory or not.
r	:	Failed automatic unit is under repair.
q_{ij} (t) / Q_{ij}(t)	:	p.d.f and c.d.f of first passage time from regenerative state i to regenerative state j.
P_{ij}(t)	:	Steady state transition probability from state S_i to S_j.
μ_i	:	Mean sojourn time in state S_i.

Table 1: "Nomenclature"

Up States

$S_0 \equiv (A_O, A_O)$, $S_1 \equiv (A_i, A_O)$, $S_3 \equiv (A_r, A_O)$,

Down State

$S_2 \equiv (A_i, A_{iw})$, $S_4 \equiv (A_r, A_i)$ $S_5 \equiv (A_r, A_{rw})$

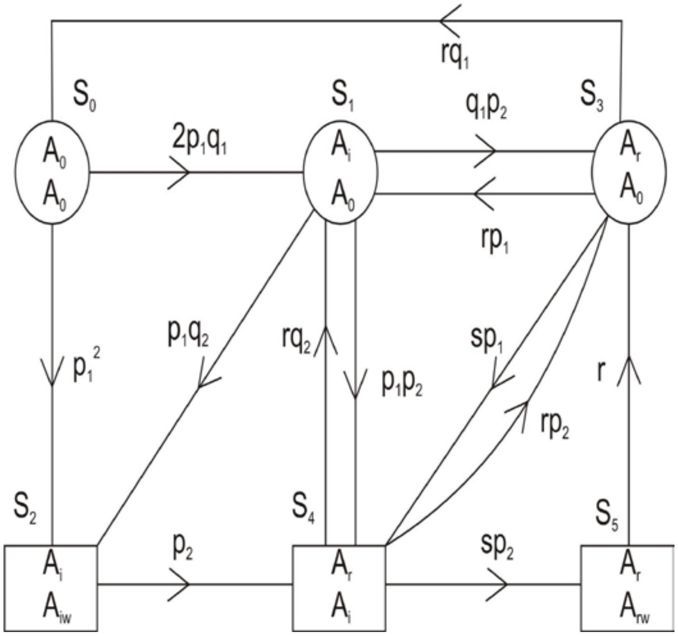

Figure-1: "Transition Diagram"

4 Transition Probabilities and Sojourn Times

Q_{01} (t) = $\dfrac{2p_1q_1[1-q_1^{2(t+1)}]}{1-q_1^2}$ Q_{02} (t) = $\dfrac{p_1^2[1-q_1^{2(t+1)}]}{1-q_1^2}$

Q_{12} (t) = $\dfrac{p_1q_2[1-q_1q_2^{(t+1)}]}{1-q_1q_2}$ Q_{13} (t) = $\dfrac{q_1p_2[1-q_1q_2^{(t+1)}]}{1-q_1q_2}$

Q_{14} (t) = $\dfrac{p_1p_2[1-q_1q_2^{(t+1)}]}{1-q_1q_2}$ Q_{24}(t) = $\dfrac{p_2[1-q_2^{(t+1)}]}{1-q_2}$

Q_{30} (t) = $\dfrac{rq_1[1-sq_1^{(t+1)}]}{1-sq_1}$ Q_{31} (t) = $\dfrac{rp_1[1-sq_1^{(t+1)}]}{1-sq_1}$

Q_{34} (t) = $\dfrac{sp_1[1-sq_1^{(t+1)}]}{1-sq_1}$ Q_{41}(t) = $\dfrac{rq_2[1-sq_2^{(t+1)}]}{1-sq_2}$

Q_{43}(t) = $\dfrac{rp_2[1-sq_2^{(t+1)}]}{1-sq_2}$ Q_{45}(t)= $\dfrac{sp_2[1-sq_2^{(t+1)}]}{1-sq_2}$

Q_{53}(t) = $\dfrac{r[1-s^{(t+1)}]}{1-s}$ (1-13)

The steady state transition probabilities from state S_i to S_j can be obtained from

$$P_{ij} = \lim_{t\to\infty} Q_{ij}$$

It can be verified that

$P_{01} + P_{02} = 1$, $P_{12} + P_{13} + P_{14} = 1$, $P_{24} = 1$, $P_{30} + P_{31} + P_{34} = 1$, $P_{41} + P_{43} + P_{45} = 1$, $P_{53} = 1$.

(13-19)

5 Mean Sojourn Times

Let T_i be the sojourn time in state S_i (i = 0, 1, 2, 3, 4, 5), then mean sojourn time in state S_i is given by

$$\mu_i = E(T_i) = \sum_{t=0}^{\infty} P(T_i > t)$$

so that

$\mu_0 = \dfrac{1}{1-q_1^2}$, $\mu_1 = \dfrac{1}{1-q_1q_2}$, $\mu_2 = \dfrac{1}{1-q_2}$,

$\mu_3 = \dfrac{1}{1-sq_1}$ $\mu_4 = \dfrac{1}{1-sq_2}$, $\mu_5 = \dfrac{1}{1-s}$

Mean sojourn time (m_{ij}) of the system in state S_i when the system is to transit into S_j is given by

$$m_{ij} = \sum_{t=0}^{\infty} t\, q_{ij}(t)$$

$m_{01} + m_{02} = q_1^2\mu_0$, $m_{12} + m_{13} + m_{14} = q_1 q_2\mu_1$,

$m_{24} = q_2\, \mu_2$, $m_{30} + m_{31} + m_{34} = sq_1\, \mu_3$

$m_{41} + m_{43} + m_{45} = sq_2\, \mu_4$, $m_{53} = s\, \mu_5$.

6 Reliability and Mean Time to System Failure

Let $R_i(t)$ be the probability that system works satisfactorily for atleast t epochs 'cycles' when it is initially started from operative regenerative state S_i (i = 0, 1, 3).

$R_0(t) = Z_0(t) + q_{01}(t-1) © R_1(t-1).$

$R_1(t) = Z_1(t) + q_{13}(t-1) © R_3(t-1).$

$R_3(t) = Z_3(t) + q_{30}(t-1) © R_0(t-1) + q_{31}(t-1) © R_1(t-1).$ (14-16)

Taking geometric transformation on both sides, we get

$$R_0(h) = \frac{N_1(h)}{D_1(h)}$$

The mean time to system failure is

$$\mu_i = \lim_{h\to 1} \frac{N_1(h)}{D_1(h)} - 1 = \frac{N_1}{D_1}$$

where

$N_1 = \mu_0 (1- P_{13} P_{31}) + P_{01} (\mu_1 + \mu_3 P_{13}).$ (17)

$D_1 = 1- P_{13}P_{31} - P_{01}P_{13} P_{30}.$ (18)

7 Availability Analysis

Let A_i (t) is the probability that the system is up at epoch t when it is initially started from regenerative state S_i. By simple probabilistic argument the following recurrence relations are obtained.

$A_0(t) = Z_0(t) + q_{01}(t-1)© A_1(t-1) + q_{02}(t-1)© A_2(t-1).$

$A_1(t) = Z_1(t) + q_{12}(t-1)© A_2(t-1) + q_{13}(t-1)© A_3(t-1) + q_{14}(t-1)© A_4(t-1).$

$A_2(t) = q_{24}(t-1)© A_4(t-1).$

$A_3(t) = Z_3(t) + q_{30}(t-1)© A_0(t-1) + q_{31}(t-1)© A_1(t-1) + q_{34}(t-1)© A_4(t-1).$

$A_4(t) = q_{41}(t-1)© A_1(t-1) + q_{43}(t-1)© A_3(t-1)+ q_{45}(t-1)© A_5(t-1).$

$A_5(t) = q_{53}(t-1)© A_3(t-1).$ (19-24)

By taking geometric transformation and solving the equation

$$A_0(h) = \frac{N_2(h)}{D_2(h)}$$

and Z_i (h) = μ_i

The steady state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$A_0 = - \frac{N_2(1)}{D'_2(1)}$$

where

$N_2(1) = (\mu_3 + \mu_0 P_{30}) (1- P_{41} + P_{41}P_{13}) + \mu_1[P_{01} (1- P_{34} + P_{34}P_{41}) + P_{02} (P_{41} + P_{31} - P_{31}P_{41})].$ (25)

$D'_2(1) = -\{(\mu_3 + \mu_0 P_{30}) (1- P_{41} + P_{41}P_{13}) + \mu_1[P_{01} (1- P_{34} + P_{34}P_{41}) + P_{02} (P_{41} + P_{31} - P_{31}P_{41})] + \mu_2\{P_{01}P_{12}(1- P_{34} + P_{34}P_{41}) + P_{02}[P_{30}(1- P_{14}P_{41}) + P_{12}(P_{31} + P_{34}P_{41})]\} + (\mu_4 + \mu_5 P_{45}) (1- P_{13}P_{31} - P_{13}P_{30}P_{01})\}$ (26)

8 Busy Period Analysis

8.1 Case-I: Busy period of Inspector

Let B_i (t) be the probability of the inspector who inspect the failed unit before being repaired by repairman. Using simple probabilistic arguments, as in case of reliability and availability analysis the following recurrence relations can be easily developed.

$B_0(t) = q_{01}(t-1)© B_1(t-1) + q_{02}(t-1)© A_2(t-1).$

$B_1(t) = Z_1(t) + q_{12}(t-1)© B_2(t-1) + q_{13}(t-1)© B_3(t-1) + q_{14}(t-1)© B_4(t-1).$

$B_2(t) = Z_2(t) + q_{24}(t-1)© B_4(t-1).$

$B_3(t) = q_{30}(t-1)© B_0(t-1) + q_{31}(t-1)© B_1(t-1)+ q_{34}(t-1)© B_4(t-1).$

$B_4(t) = Z_4(t) + q_{41}(t-1)© B_1(t-1) + q_{43}(t-1)© B_3(t-1)+ q_{45}(t-1)© B_5(t-1).$

$B_5(t) = q_{53}(t-1)© B_3(t-1).$

(27-32)

By taking geometric transformation and solving the equation

$$B_0(h) = \frac{N_3(h)}{D_2(h)}$$

The probability that the inspection facility is busy in inspecting the failed unit is given by

$$B_0 = \lim_{t\to\infty} B_0(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$B_0 = -\frac{N_3(1)}{D_2'(1)}$$

where

$$N_3(1) = \mu_1\{P_{01}(1-P_{34}+P_{34}P_{41}) + P_{02}(P_{41}+P_{31}-P_{31}P_{41})\} + \mu_2\{P_{01}P_{12}(1-P_{34}+P_{34}P_{41}) + P_{02}[P_{30}(1-P_{14}P_{41}) + P_{12}(P_{31}+P_{34}P_{41})]\} + \mu_4(1-P_{13}P_{31}-P_{13}P_{30}P_{01}).$$

(33)

and $D_2'(1)$ is the same as in availability analysis.

8.2 Case-II: Busy period of Repairman

Let $B_i'(t)$ be the probability that the repair facility is busy in repair of failed unit when the system initially starts from regenerative state S_i. Using simple probabilistic arguments, the following recurrence relations can be easily developed.

$$B_0'(t) = q_{01}(t-1)© B_1'(t-1) + q_{02}(t-1)© B_2'(t-1).$$

$$B_1'(t) = q_{12}(t-1)© B_2'(t-1) + q_{13}(t-1)© B_3'(t-1) + q_{14}(t-1)© B_4'(t-1).$$

$$B_2'(t) = q_{24}(t-1)© B_4'(t-1).$$

$$B_3'(t) = Z_3(t) + q_{30}(t-1)© B_0'(t-1) + q_{31}(t-1)© B_1'(t-1) + q_{34}(t-1)© B_4'(t-1).$$

$$B_4'(t) = Z_4(t) + q_{41}(t-1)© B_1'(t-1) + q_{43}(t-1)© B_3'(t-1) + q_{45}(t-1)© B_5'(t-1).$$

$$B_5'(t) = Z_5(t) + q_{53}(t-1)© B_3'(t-1).$$

(34-39)

By taking geometric transformation and solving the equation

$$B_0'(h) = \frac{N_4(h)}{D_2(h)}$$

The probability that the repair facility is busy in repair of failed unit is given by

$$B_0' = \lim_{t\to\infty} B_0'(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$B_0' = -\frac{N_3(1)}{D_2'(1)}$$

where

$$N_4(1) = \mu_3(1-P_{41}+P_{41}P_{13}) + (\mu_4+\mu_5P_{45})(1-P_{13}P_{31}-P_{13}P_{30}P_{01}).$$

(40)

and $D_2'(1)$ is the same as in availability analysis.

9 Profit Function Analysis

The expected total profit in steady-state is

$$P = C_0A_0 - C_1B_0 - C_2B_0' \qquad (41)$$

where

C_0: be the per unit up time revenue by the system

C_1 & C_2: be the per unit down time expenditure on the system

10 Graphical Interpretation

The behaviour of the MTSF and the profit function w.r.t failure rate and repair rate have been studied through graphs by fixing the values of certain parameters C_0, C_1 and C_2 as $C_0 = 2000$, $C_1 = 100$ and $C_2 = 500$.

On the basis of the numerical values taken as:

$r = 0.5$, $p_1 = 0.2$ and $p_2 = 0.8$

The values of various measures of system effectiveness are obtained as:

Mean time to system failure (MTSF) = 4.1201466

Availability (A_0) = 0.639677

Busy period of Inspector (B_0) = 0.394281

Busy period of repairman (B_0') = 0.63085

Profit (P) = 924.50161

Figure: 2 show the behavior of MTSF w.r.t failure rate (p_1) for different values of repair rate (r).It appears from graph that MTSF decreases with increase in failure rate.

Figure: 3 show the behavior of MTSF w.r.t repair rate (r) for different values of failure rate (p_1).It appears from graph that MTSF increases with increase in repair rate.

Figure: 4 show the behavior of Profit function w.r.t failure rate (p_1) for different values of repair rate (r). It appears from graph that Profit decreases with increase in failure rate.

Figure: 5 show the behavior of Profit function w.r.t repair rate (r) for different values of failure rate (p_1). It appears from graph that Profit increases with increase in repair rate.

11 Conclusion

This paper conclude by providing the results for the various reliability measures like MTSF, availability and busy period of repairman and inspector that the availability of system is increased by proper maintenance of units.This leads to increase the profit of the system. It also provides information for other researchers and companies following such systems to prefer the equipments which satisfied the conditions as discussed in this paper.

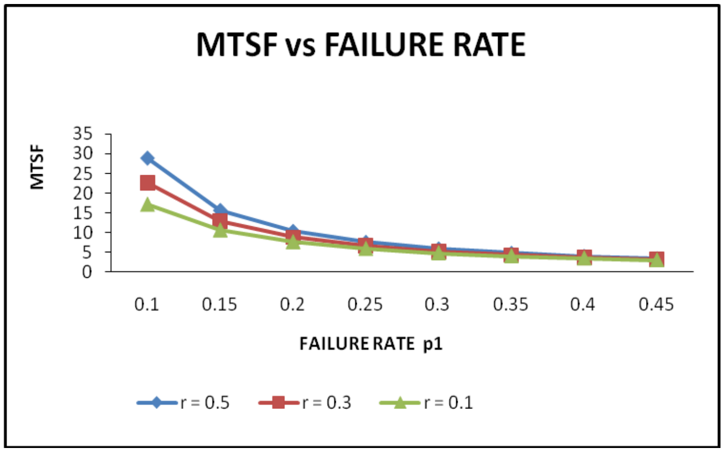

Figure 2: "MTSF vs FAILURE RATE"

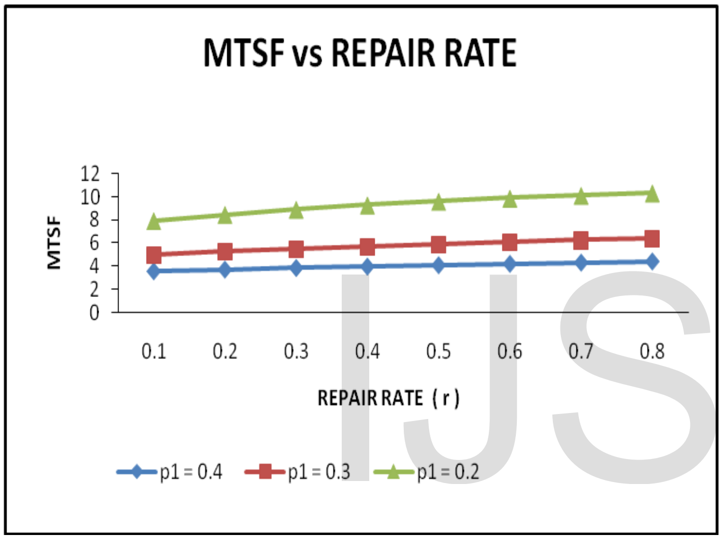

Figure 3: "MTSF vs REPAIR RATE"

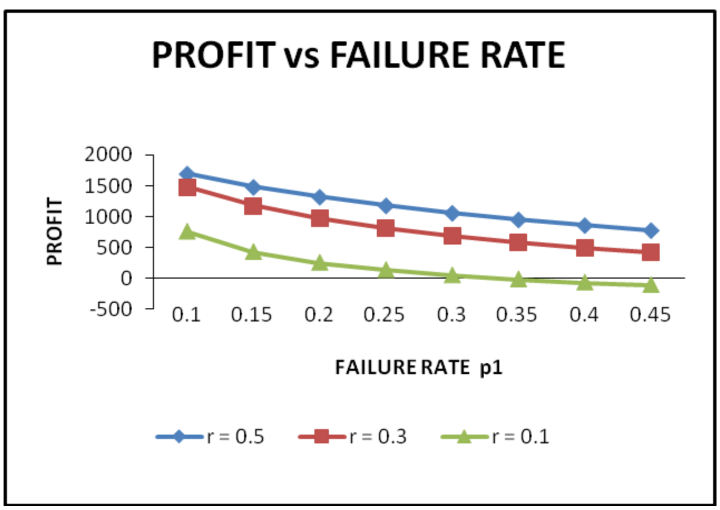

Figure 4: "PROFIT vs FAILURE RATE"

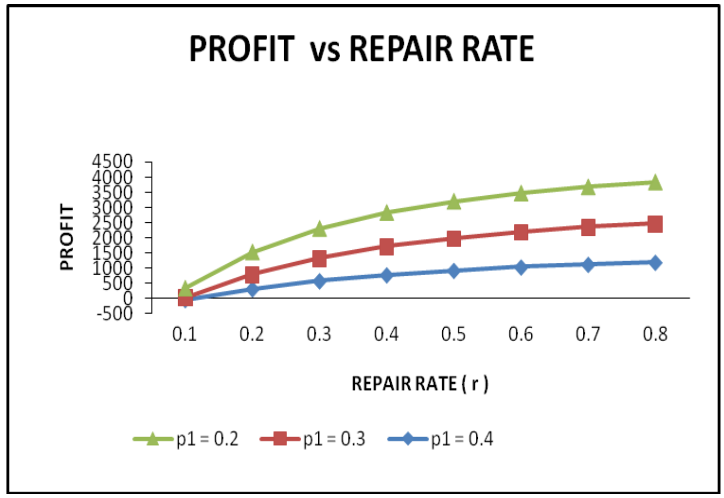

Figure 5: "PROFIT vs REPAIR RATE"

REFERENCES

1. **Bhardwaj N (2009)** "Analysis of two unit redundant system with imperfect switching and connection time", International transactions in mathematical sciences and Computer, July-Dec. 2009,Vol. 2, No. 2, pp. 195-202.
2. **Ram Kishan, Manish Kumar (2009)**, "Stochastic analysis of a two-unit parallel system with preventive maintenance", Journal of Reliability and Statistical Studies (ISSN: 0974-8024), Vol. 2, Issue 2(2009): 31-38.
3. **Bhardwaj N, Kumar A and Kumar S (2008)** "Stochastic analysis of a single unit redundant system with two kinds of failure and repairs", Reflections des. ERA-JMS, Vol. 3 Issue 2 (2008), 115-134.
4. **Gupta R, Varshney G (2007)** "A two identical unit parallel syatem with Geometric failure and repair time distributions", J. of comb. Info. & System Sciences, Vol. 32, No.1-4, pp 127-136 (2007).
5. **Padgett, Spurrier (1985),** "On discrete failure models", IEEE Transactions on Reliability, Vol. R-34, No. 3, 1985.
6. **Said K.M.EL, Salah M, Sherbeny EL (2005)** "Profit analysis fof a two unit cold standby system with preventive maintenance and random change in Units", 1(1):71-77, 2005, ISSN 1549-3644 (2005).
7. **Taneja G, Tyagi V K, Bhardwaj P (2004)** "Profit analysis of a single unit programmable logic controller (PLC)", Pure Appl. Math. Sci., vol. LX 1-2, September 2004.
8. **Goel, L.R., G.C.Sharma, P.Gupta (1986)**, "Reliability analysis of a system with preventive maintenance inspection and two types of repair", Micoelect. Reliability,26: 429-433.
9. **Naidu.R.S, Gopalan.M.N (1984)**, "On the stochastic behavior of a one-server two unit system subject to

arbitrary failure, random inspection and failure modes", Microelectronics 7 reliability,vol.24.1984,pp 375-378.

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